NONSTEADY-STATE NONISOTHERMAL FLOW OF A MAGNETIC LIQUID IN A PLANE CHANNEL

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Magnetic liquids are finding wider and wider use in various fields of technology [1]. Such liquids can be used as heat exchange fluids in equipment which generates a magnetic field under conditions of weightlessness [2] and in a number of other applications. The efficiency of heat exchange equipment is determined to a significant degree by the temperature of the magnetic liquid. In connection with this fact, it is of interest to examine nonisothermal flows at a temperature near the Curie point, where the dependence of volume magnetization M on temperature is expressed most clearly. In this case the character of the liquid flow will be affected not only by the dependence of saturation volume magnetization on temperature, but also by temperature inhomogeneity caused by development of external heat sources and sinks produced by the magnetocaloric effect. We note that although this is a weak effect [3], the temperature redistribution over channel section which it produces may be significant. With a high gradient in the external magnetic field H even a small change in temperature can significantly change the force acting on a magnetic liquid element. The unique features of magnetic liquid flow at a temperature close to the Curie point can be investigated by simultaneously solving the equations of motion and thermal conductivity.

In the Rosensweig–Neuringer approximation [4] the system of equations describing an incompressible nonconductive saturated magnetic liquid has the form

$$\rho\left[\frac{\partial \mathbf{v}}{\partial t}+(\mathbf{v}\nabla)\mathbf{v}\right]=-\nabla p+\eta\Delta\mathbf{v}+\mu_0 M\nabla H; \tag{1}$$

$$\rho c\left[\frac{\partial T}{\partial t}+\mathbf{v}\nabla T\right]=\lambda\Delta T-\mu_0\left(\frac{\partial M}{\partial T}\right)_{\rho,H}T\left[\frac{\partial H}{\partial t}+\mathbf{v}\nabla H\right],\quad \operatorname{div}\mathbf{v}=0,\ M=M(T), \tag{2}$$

where ρ is the density; $\mathbf{v}$ is the velocity vector, t, time; p, pressure; η, dynamic viscosity; μ_0, magnetic permittivity of a vacuum; c, specific heat; T, temperature; λ, thermal conductivity. Below we will assume that $M = \Lambda(T_c - T)$, where $\Lambda = -(\partial M/\partial T)_{\rho,H}$ is the pyromagnetic coefficient and T_c is the Curie temperature [4, 5]. Since the change in temperature is small in comparison to the absolute temperature, in the last term of Eq. (2) which considers the magnetocaloric effect, we may assume T = const.

Within a planar channel of width 2L let a magnetic liquid move under the influence of gradients in external magnetic field $G = \partial H/\partial x$ = const and pressure $\partial p/\partial x$ = const. The channel wall temperature T_0 is maintained constant and close to the Curie point. We choose as characteristic values for length L, for time $L^2\rho/\eta$, for temperature $(T_c - T_0)$, and for velocity $(T_c - T_0)\sqrt{\lambda/(\eta T_0)}$. Then the system (1), (2) describing one-dimensional flow of the magnetic liquid in the planar channel can be written in dimensionless form

$$\partial u/\partial t = \partial^2 u/\partial y^2 - 2k^2\theta + q; \tag{3}$$

$$\Pr\,\partial\theta/\partial t = \partial^2\theta/\partial y^2 + 2k^2 u, \tag{4}$$

where $\theta = (T - T_0)/(T_c - T_0)$; $2k^2 = \mu_0\Lambda G L^2\sqrt{T_0/\eta\lambda}$ $(k > 0)$; $q = 2k^2 - P$; $P = [L^2/(T_c - T_0)] \times \sqrt{T_0/\eta\lambda}\cdot\partial p/\partial x$; $\Pr = \eta c/\lambda$ is the Prandtl number; u – x is the projection of the velocity vector. The y axis is directed perpendicular to the channel walls and the x axis, along the channel axis.

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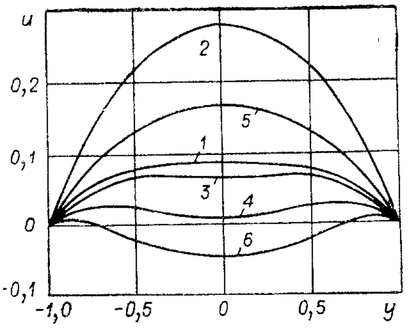

Fig. 1

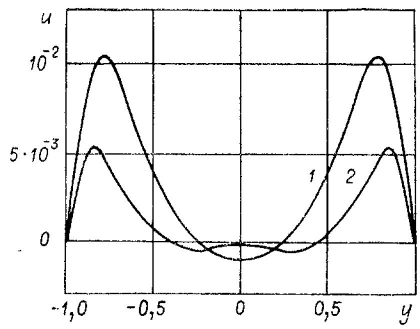

Fig. 2

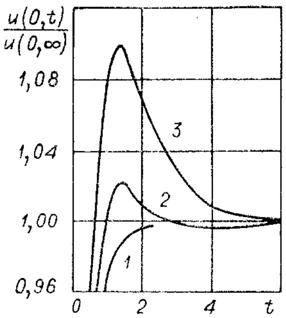

Fig. 3

We will consider the developing flow of a magnetic liquid. At time $t < 0$ let $q = 0$, i.e., magnetic pressure compensates the hydrodynamic pressure head in the channel and the liquid is at rest (magnetic stoppage). At the initial moment ($t = 0$) an instantaneous change occurs in pressure gradient, so that at $t \geqslant 0$ the quantity q will have a constant nonzero value. The corresponding initial and boundary conditions have the form

$$u(y,\ 0) = 0,\quad \theta(y,\ 0) = 0,\quad u(\pm 1,\ t) = 0,\quad \theta(\pm 1,\ t) = 0. \tag{5}$$

Using a Laplace transform with respect to the variable t, we reduce the solution of Eqs. (3)-(5) to solution of a system of ordinary differential equations with boundary conditions [6]. Then performing the inverse transform, we obtain

$$u(y,\ t) = \frac{q}{2k^2}\,\frac{\operatorname{sh} k \sin k \operatorname{ch}(ky) \cos(ky) - \operatorname{ch} k \cos k \operatorname{sh}(ky) \sin(ky)}{\operatorname{sh}^2 k + \cos^2 k}$$

$$+\ 2q \sum_{j=0}^{\infty} \exp\left(-\frac{\mathrm{Pr}+1}{2\,\mathrm{Pr}}\, n^2 t\right) \frac{(-1)^j \cos(ny)}{n(n^4 + 4k^4)} \left[\frac{n^4(\mathrm{Pr}-1) + 8k^4\,\mathrm{Pr}}{\alpha} \operatorname{sh}\left(\frac{\alpha t}{2\,\mathrm{Pr}}\right) - n^2 \operatorname{ch}\left(\frac{\alpha t}{2\,\mathrm{Pr}}\right)\right], \tag{6}$$

where $n = \pi(j + 0.5)$; $\alpha = \sqrt{n^4(\mathrm{Pr} - 1)^2 - 16k^4\mathrm{Pr}}$. An expression for $\theta(y,\ t)$ can be obtained from Eq. (3). Since at the Curie temperature the liquid loses its ferromagnetic properties the assumed dependence $M = M(T)$ is not followed at $T > T_c$; consequently the solution obtained, Eq. (6), is valid only for $\theta(y,\ t) \leqslant 1$, which in turn imposes limitations on the parameters Pr, k, and q.

Velocity profiles calculated with Eq. (6) for $\mathrm{Pr} = 5$, $q = 1$ are shown in Fig. 1, where curves 1-3 correspond to $k = 0.5\pi$, with 4-6 representing $k = 0.75\pi$. Curves 1, 4 are steady-state velocity profiles in the channel, while 2, 3, 5, 6 are profiles at times $t = 0.5$, 2, 0.3, and 1. For comparison, Fig. 2 shows steady-state velocity profiles for $q = 1$, $k = 1.25\pi$ (curve 1) and $k = 1.75\pi$ (curve 2).

Analysis of the solution obtained shows that the transient process of establishing velocity and temperature profiles in the channel may be periodic or aperiodic, depending on the relationship between the parameters Pr and k. At $16k^4\mathrm{Pr} > (0.5\pi)^4(\mathrm{Pr} - 1)^2$ the transient process will be periodic (damping oscillations) while at $16k^4\mathrm{Pr} \leqslant (0.5\pi)^4(\mathrm{Pr} - 1)^2$ it is aperiodic. Figure 3 shows the ratio $u(0,\ t)/u(0,\ \infty)$ as a function of time for parameters $q = 1$, $k = 0.25\pi$. Curves 1-3 correspond to $\mathrm{Pr} = 0.3, 1, 3$. With increase in Prandtl number the maximum amplitude of the deviation of $u(y,\ t)$ from $u(y,\ \infty)$ increases in the transient process, while the time required to establish a steady-state profile also increases.

By integrating the function $u(y,\ \infty)$ over channel section, we obtain an expression for the fluid expenditure in the steady-state flow. The dependence of flow rate Q on the parameter k is shown in Fig. 4 for $P = 0.5\pi$ and 0.75π (curves 1, 2). The functions obtained show that at a certain value of the parameter k the flow rate of the magnetic liquid in the channel is at a maximum. Further increase in external magnetic field gradient (increase in k) leads to a decrease in flow rate.

We will now consider steady-state oscillations of the velocity and temperature profiles in the channel under the action of a constant external magnetic field gradient and a pressure gradient which varies periodically by a law $P = 2k^2 - a\exp(i\omega t)$, where a is the dimensionless amplitude and ω is the dimensionless frequency. Substituting this expression for P in Eq. (3) and taking the transform $u = U\exp(i\omega t)$ and $\theta = \Theta\exp(i\omega t)$, we transform Eqs. (3), (4)

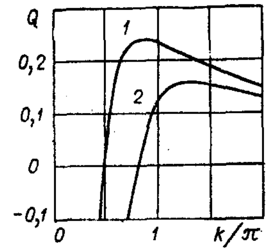

Fig. 4

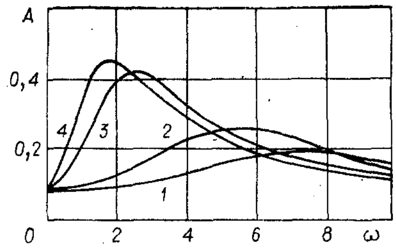

Fig. 5

to a system of ordinary differential equations in U = U(y) and $\Theta = \Theta(y)$ with boundary conditions

$$U(\pm 1) = 0, \quad \Theta(\pm 1) = 0.$$

Finally, we obtain

$$u(y, t) = \frac{ia \exp(i\omega t)}{4k^4 - \omega^2 \Pr}\left[\omega \Pr - \frac{\omega \Pr \beta - \omega^2 \Pr(\Pr - 1) - 8k^4}{2\beta} \times \frac{\operatorname{ch}(s_1 y)}{\operatorname{ch} s_1} - \frac{\omega \Pr \beta + \omega^2 \Pr(\Pr - 1) + 8k^4}{2\beta}\frac{\operatorname{ch}(s_2 y)}{\operatorname{ch} s_2}\right], \tag{7}$$

where

$$\beta = \sqrt{\omega^2(\Pr - 1)^2 + 16k^4}; \quad s_1 = \sqrt{\frac{i\omega(\Pr + 1) + i\beta}{2}}; \quad s_2 = \sqrt{\frac{i\omega(\Pr + 1) - i\beta}{2}}.$$

An expression for $\theta(y, t)$ can be obtained from Eq. (3). Just like Eq. (6), Eq. (7) is valid given the condition $|\theta(y, t)| \leqslant 1$. The solution obtained, Eq. (7), is of complex form, and we will consider the real and imaginary components separately. We will note that in the particular case $4k^4 - \omega^2 \Pr = 0$, Eq. (7) can be written in the form

$$u(y, t) = \frac{ia \exp(i\omega t)}{\omega(\Pr + 1)}\left[\frac{\operatorname{ch}(y\sqrt{i\omega(\Pr + 1)})}{\operatorname{ch}(\sqrt{i\omega(\Pr + 1)})} - 1\right],$$

i.e., the velocity amplitude does not become infinite. Figure 5 shows the dependence of steady-state velocity oscillation amplitude A on frequency ω, calculated with Eq. (7) for the channel section y = 0 and parameters k = 0.5π, a = 1, and Pr = 0.5, 1, 5, 10 (curves 1-4) The curves show the effects of resonance. With increase in Prandtl number the resonant frequency decreases, while the oscillation amplitude increases.

The dimensionless parameters used in the calculations correspond to physical quantities within the range L = 0.01-0.1 m, T_0 = 300-400°K, Λ = 100-200 A/(m·deg), G = 10^4-10^5 A/m^2, η = 10^{-3}-10^{-2} kg/(m·sec), λ = 0.1-1.0 W/(m·deg), c = 500-2000 J/(kg·deg).

Couette flow of a magnetic liquid at temperatures near the Curie point can be studied in a similar manner. We note that one characteristic feature will then be the appearance of reverse flows near the nonmoving wall.

In conclusion, we note that unique features found in magnetic liquid flow are caused by the temperature dependence of volume magnetization and the magnetocaloric effect.

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STATIONARY FLOW OF A REACTING LIQUID WHOSE PROPERTIES VARY WITH THE EXTENT OF REACTION

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Considerable interest attaches to the flow of a reacting liquid whose properties change during the reaction in relation to analysis of displacement-type flow polymerization reactors. There is a substantial increase in viscosity as the polymerization proceeds (by up to a factor 10^6 or more), which produces qualitative changes in the flow picture, and this in turn influences the macrokinetic relationships.

Here we consider the simple case of isothermal flow of a reacting liquid in which the extent of reaction and the properties are uniquely determined by the reaction time. A general self-modeling solution is derived and the main features of the flow are examined for the case where there is a considerable increase in viscosity.

1. Consider the stationary laminar flow of a reacting Newtonian liquid in a tube (tubular flow reactor). The viscosity μ and density ρ alter from the initial values μ_0 and ρ_0 at the inlet to the final values μ_1 and ρ_1 on complete reaction. The temperature is taken as constant, and the reactions are independent of the velocity gradients, while the effects of diffusion are neglected because of the smallness of the diffusion coefficients. The extent of reaction and the properties of the liquid are uniquely determined by the reaction time t, and the relationships are considered as given.

To derive the flow pattern we assume that the radial velocity component arising from change in the flow profile on account of the change in properties is small by comparison with the axial component, while the pressure change along the radius is slight, and also that the viscosity is large enough for one to neglect inertia and the effects of the inlet hydrodynamic-stabilization part. With these assumptions, the flow at each section is essentially plane-parallel, which is an approximation widely used in various applications to the flow of liquids with varying properties [1-3]. The general equations of motion for a Newtonian liquid [4] in this approximation give

$$\frac{1}{R}\frac{\partial}{\partial R}\left(\mu R\frac{\partial V}{\partial R}\right)+\frac{dP}{dZ}=0,\quad 0\leqslant R\leqslant R_0,\quad 0<Z<Z_0, \tag{1.1}$$

where V is the axial component of the flow velocity, R is the distance from the axis, P = P(Z) is the difference between the pressure at the inlet and that in a given section, Z is the distance from the start, and Z_0 and R_0 are the tube length and radius correspondingly. The radial velocity component W is given by the equation of continuity

$$\frac{1}{R}\frac{\partial}{\partial R}(\rho RW)+\frac{\partial}{\partial Z}(\rho V)=0. \tag{1.2}$$

The reaction time t is the time from the instant when an element of the liquid enters the reactor and is given by

$$t=\int\limits_0^Z\frac{dz}{V}, \tag{1.3}$$

in which the integration is carried out along the path of motion of that element, i.e., along a given flow line $\psi(Z, R)$ = const, where

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